



# Using Virtual Manipulatives to Teach Mathematics

## Center for Technology Implementation

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### Overview

Using manipulatives in mathematics instruction can help you to address the Common Core State Standards for mathematical practice.

Abstract concepts are essential to understanding and doing mathematics. They are also a source of difficulty for students who struggle with mathematics, many of whom find even basic mathematics concepts hard to understand. Teachers often use tangible materials—referred to as manipulatives—to concretely represent these abstract concepts and help their students understand them. Manipulatives can also be used to help students link these concepts with prior knowledge. Using manipulatives in mathematics instruction can help you to address the [Common Core State Standards for mathematical practice](#), particularly the standards related to [making sense of problems](#) and [abstract reasoning](#).

In the past, most classrooms only had physical manipulatives. Today, however, more classrooms have improved access to computers and the Internet, and virtual manipulatives are becoming increasingly common. Virtual manipulatives can be useful tools for students, and they can help them learn how to use [appropriate technology tools](#) for mathematics.

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### In Your Classroom

Virtual manipulatives are digital “objects” that resemble physical objects.

You can manipulate these digital objects, often with a mouse, just as you would manipulate physical objects. Many of the virtual manipulatives typically used in mathematics education are [available](#) for [free online](#). These include Base-10 Blocks, Cuisenaire Rods, and tangrams. Most virtual manipulatives come with structured activities or suggestions to help teachers use them in the classroom.

### Benefits of Virtual Manipulatives

Virtual manipulatives can benefit your students in several ways. Using these tools in your classroom can:

- ▶ Help students understand abstract mathematics concepts
- ▶ Lead to a richer and more complex understanding of concepts
- ▶ Help clarify student misconceptions and build connections between concepts and representations

These tools can be especially helpful for students with disabilities because they can improve their understanding of the abstract symbolic language of mathematics. Students who struggle in mathematics often find it hard to connect visual and symbolic representations, but virtual manipulatives can help make these connections clear.

In addition, virtual objects can be altered in ways that concrete ones cannot (for example, the size, shape, and color of a block can be changed). In many cases, this enables students to create more examples than they could with physical objects. Many virtual manipulatives also have the added benefit of providing students with hints and feedback, allowing them to practice on their own without teacher assistance.

These features mean that virtual manipulatives can be especially helpful for students with language difficulties, including English language learners. These students often have trouble explaining what they are learning in mathematics, but virtual manipulatives can help them clarify their thinking and share their thoughts with others.

## Choosing Virtual Manipulatives

There are many virtual manipulatives to choose from. When deciding which ones to use in your classroom, remember to:

- ▶ Ensure that the manipulatives are linked to specific mathematics content and evidence-based strategies (and that they are not used as a separate activity).
- ▶ Ensure that the manipulatives are used as part of a progression from concrete to pictorial to abstract (see the section on What the Research Says for more detail).
- ▶ Evaluate embedded options and supports:
  - Can you adjust the level of difficulty for different students?
  - Is feedback provided? If so, what type of feedback?
  - Will you need to be on hand to provide additional feedback and support?
  - How clear are the instructions for use? Will you need to provide additional guidance?

Used wisely, virtual manipulatives can be an excellent addition to your teaching toolkit. They can provide students with opportunities for guided exploration, helping them build a solid understanding of mathematics concepts. They can also help students demonstrate and share their learning.

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## What the Research Says

Studies have evaluated the effectiveness of manipulatives as a tool in mathematics instruction. One line of research has studied the Concrete-Representational-Abstract (CRA) sequence. This form of explicit instruction moves students from concrete manipulatives to pictorial representations of those manipulatives, and from pictorial representations to abstract concepts. Butler, Miller, Crehan, Babbitt, and Pierce (2003) compared the effectiveness of teaching fraction concepts to students with learning disabilities using a CRA approach versus a Representational-Abstract (RA) approach (which starts with pictorial representations and then moves to abstract concepts without using concrete manipulatives). Although both groups improved in terms of their understanding of fractions, the CRA group had higher scores overall than the RA group.

A study by Witzel, Mercer, and Miller (2003) also supports the effectiveness of adopting a CRA approach to develop the basic mathematics skills of students with learning disabilities. Students were taught to solve algebraic equations using either a CRA approach or a traditional approach. The study involved 34 matched pairs of students in Grades 6 and 7 who had either been diagnosed with learning disabilities or categorized as at risk for learning problems. Although both groups showed improvement after a four-week intervention, the CRA group significantly outperformed the group that had received traditional instruction.

In another CRA study (Maccini & Hughes, 2000), six adolescents with learning disabilities used algebra tiles to represent algebra word problems during the concrete phase of instruction. The students were able to transition successfully to pictorial and ultimately symbolic representations of the problems.

With the rapid development of technology, virtual manipulatives have become widely used in mathematics instruction and have been shown to have a positive impact on students' mathematics achievements. Bolyard and Moyer-Packenham (2012) conducted a quasi-experimental study to examine the effectiveness of virtual manipulatives in helping sixth-grade students make sense of integer arithmetic. The results revealed that the use of virtual manipulatives helped students improve integer computation achievement.

Reimer and Moyer (2005) investigated the performance of 19 third-grade students during a two-week unit on fractions that used virtual manipulatives. More than half of the students improved their conceptual understanding of fractions on a teacher-designed measure. In another study of 19 second-grade students, Moyer, Niezgoda, and Stanley (2005) observed that virtual Base-10 Blocks enabled students to demonstrate more sophisticated strategies and explanations of place value. Bolyard and Moyer-Packenham (2006) studied the use of virtual manipulatives with 99 sixth-grade students learning addition and subtraction of integers. The students showed significant gains in achievement, and the researchers concluded that virtual manipulatives can support learning these concepts.

Bouck, Satsangi, Doughty, and Courtney (2014) used virtual manipulatives and concrete manipulatives to teach subtraction skills to a group of elementary-aged students with autism spectrum disorder. Although both types of manipulatives helped students with mathematics problem solving, students using virtual manipulatives achieved greater accuracy and faster independence in solving the subtraction problems. Suh and Moyer (2007) compared the use of concrete and virtual manipulatives among third-grade students studying algebraic thinking. Both types of manipulatives were associated with higher achievement and increased flexibility in representing algebraic concepts. Steen, Brooks, and Lyon (2006) compared the academic achievement of a group of first-grade students who used virtual manipulatives for practice in geometry instruction (the treatment group) with another group who did not use virtual manipulatives (the control group). A total of 31 students were randomly assigned to either the treatment or control group. Achievement was measured by the Grade 1 and Grade 2 assessments provided by the classroom textbook's publisher. The treatment group improved significantly on both the Grade 1 and Grade 2 tests, while the control group showed significant improvement only on the Grade 1 test. The teacher of the treatment group also noted that her students showed increased motivation and spent more time on task.

Teachers also play an important role in helping students understand the concepts that manipulatives represent (Bjorklund, 2014; Moyer, 2001). This was highlighted in a one-year study of 10 middle school mathematics teachers and their use of manipulatives (Moyer, 2001). Teachers who were unable to represent mathematics concepts were more likely to use manipulatives as a diversionary rather than an instructional activity.

The CRA studies described above also demonstrate the importance of structure and guidance in linking concrete materials with abstract concepts. For example, students in Maccini and Hughes' study (2000) were taught not only to use algebra tiles to represent word problems, but also to use a structured strategy in solving them. In the Reimer and Moyer study (2005), students benefited from another important aspect of guidance: feedback. When students were interviewed regarding their impressions of the virtual manipulatives, an emergent theme was their appreciation of the immediate feedback provided by computer-based manipulatives. In addition, it is important for teachers to motivate students intrinsically when using manipulatives in mathematics instruction in order to achieve the desired outcomes (Jones, Uribe-Fioez, & Wilkins, 2011).

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