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Lesson in Action

Visual Representations: Geometry in Art

Context

Ms. Richardson's Grade 6 class is working on a geometry unit. They have reviewed how to classify different quadrilaterals and triangles, and most students have become proficient at using mathematical terms. They have also been learning how to use GeoGebra, and in the previous lesson they began working on decomposing polygons. Ms. Richardson would like students to continue to draw and decompose polygons in order to become more familiar with them through visualization.

Common Core State Standards

▶ CCSS Math 6.G.1 (http://www.corestandards.org/Math/Content/6/G/A/1) Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

▶ CCSS Math MP4 (http://www.corestandards.org/Math/Practice/MP4) Model with mathematics.

Lesson Objective

Students will decompose polygons into triangles, rectangles, and trapezoids.

Technology

▶ Interactive whiteboard to demonstrate students' visual representations

▶ GeoGebra to create and decompose polygons

Assessment

▶ Student discussion and final work for learning task activities

▶ Exit ticket

Ms. Richardson's Class in Action

Launch

To begin today's lesson, Ms. Richardson draws a few congruent regular pentagons using GeoGebra and displays them on the interactive whiteboard. "In the last lesson we discussed decomposing polygons. Let's have two volunteers come up and decompose these regular pentagons in two different ways." David and Emily go up to the whiteboard. "After you decompose the pentagon into smaller polygons, I'll ask you to explain how you can use the decomposition to find the area of a pentagon."
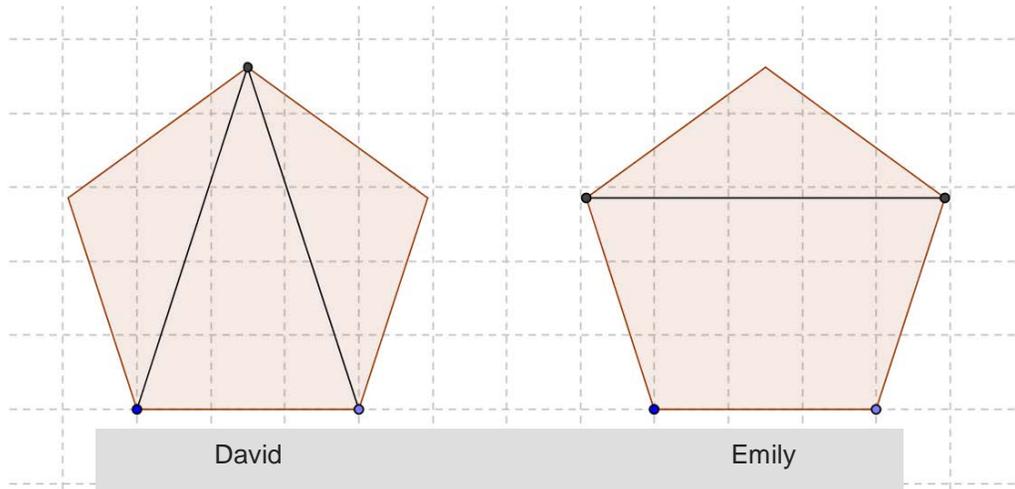

David points to his diagram. "I divided the pentagon into three triangles, because I know the formula for triangle area."

Emily points to her diagram. "I divided the pentagon into a triangle and a trapezoid, because I know the formulas for both of those shapes." Ms. Richardson asks the class, "How are the pentagon decompositions similar and different?"

Kevin raises his hand. "The triangle that Emily drew is the same as one of the triangles in David's." Maria adds, "David only used triangles and Emily used a triangle and a trapezoid." The students compare and contrast the pentagons for a few more minutes.

After concluding the discussion, Ms. Richardson asks, "Are there other ways we can divide a pentagon using only triangles?" With input from the students, she creates a few other examples:
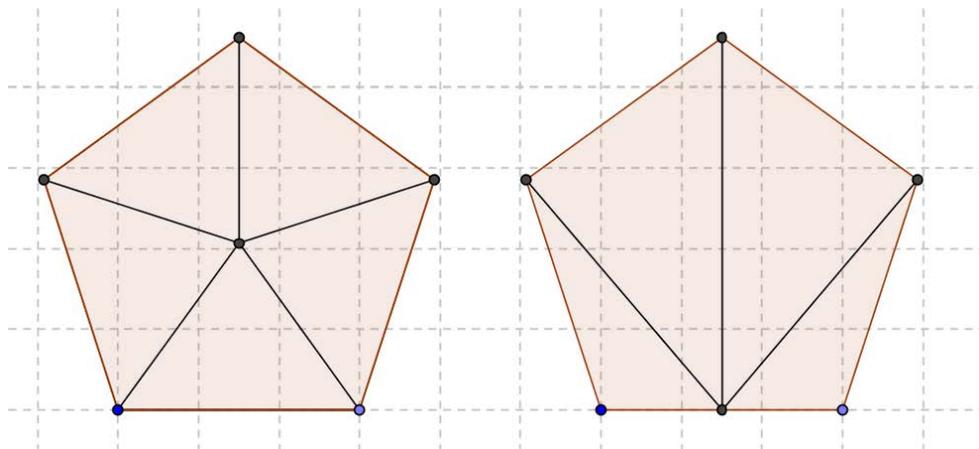
"Emily's pentagon has a triangle and a trapezoid. Is it possible to divide the pentagon in other ways with triangles and trapezoids?" For these examples, she calls on a few students to use GeoGebra and draw the line segments to decompose the pentagons.
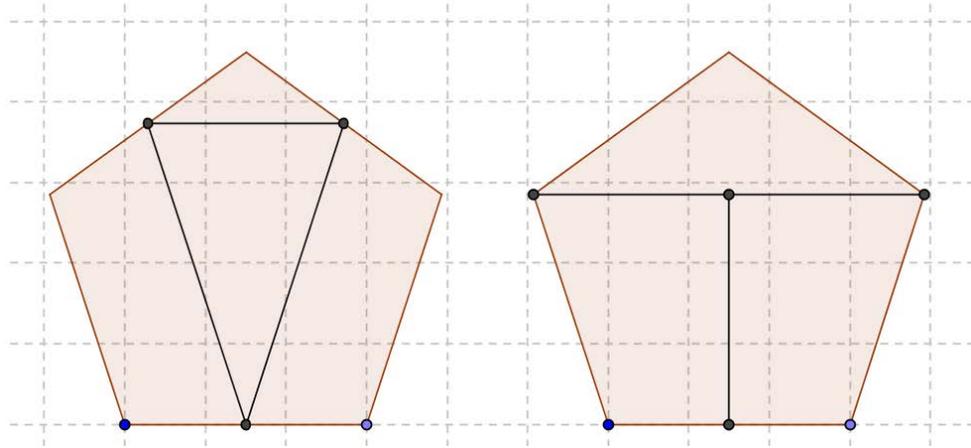
Finally, Ms. Richardson has students give a few examples with rectangles and triangles.
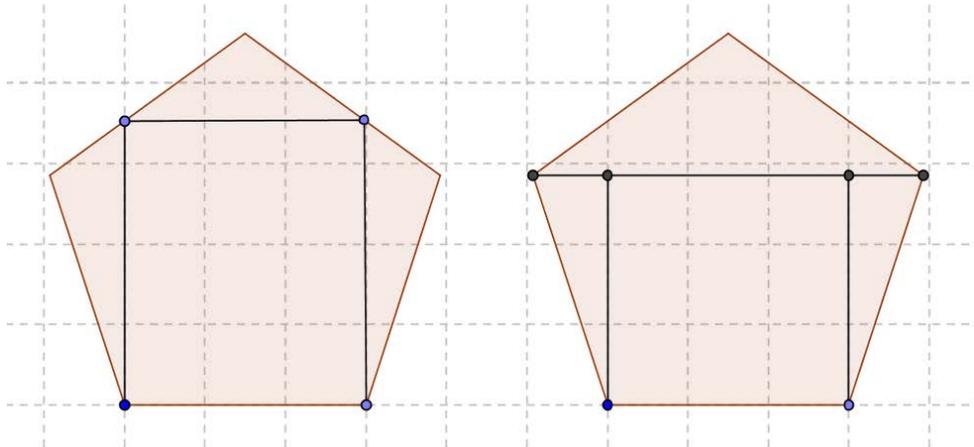
She then concludes the opening of the lesson. "Dividing the pentagons into polygons like this is pretty straightforward," she tells the class. "Next, we're going to work with a real-world example that is a little more complicated."

Learning Task

To introduce the learning task, Ms. Richardson brings up a picture of a Sol Lewitt work.


Sol Lewitt. Wall Drawing #1113. On a wall, a triangle within a rectangle, each with broken bands of color, 2003. Hirshhorn Museum and Sculpture Garden, Smithsonian Institution.

"This painting is by the artist Sol Lewitt," Ms. Richardson begins. "It's in the Hirshhorn Museum and takes up the entire wall of a room. The title of the work is actually a set of instructions." She points to the caption and tells the class, "We're going to use them to create our own versions."

She opens a new window in GeoGebra and displays it on the whiteboard. "Instead of using a physical wall," Ms. Richardson explains, "we'll model using a virtual one. First we need a triangle within a rectangle." As she is talking, she constructs a rectangle and a triangle.
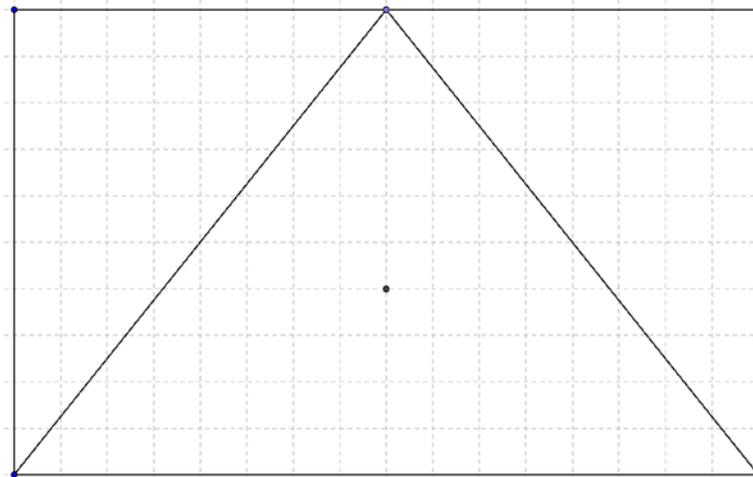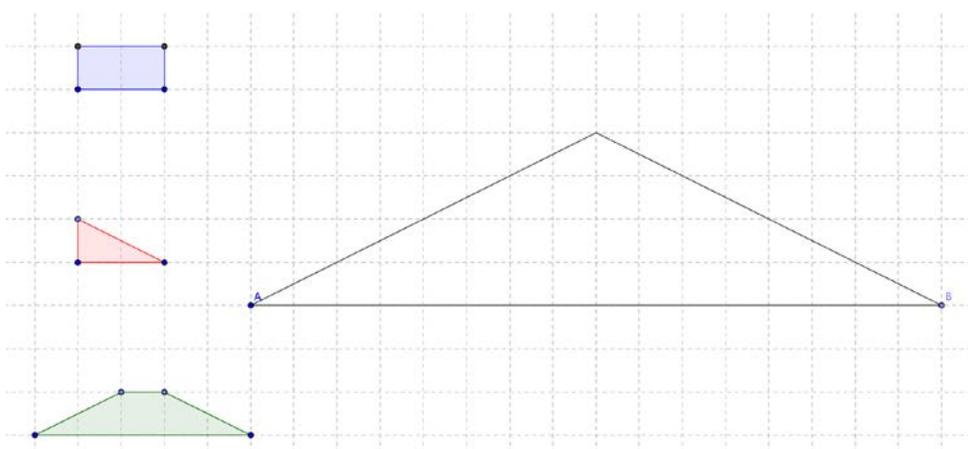
"We'll use the idea that this artist had—to divide the triangle up into three smaller triangles."
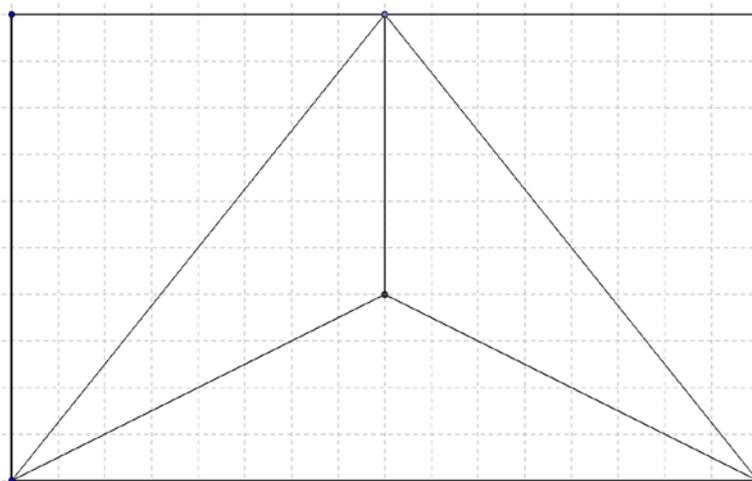
"To continue with this, you're going to work with a partner and begin constructing the bottom triangle piece."

As students get into pairs and move to the computers, she instructs them to pull up the file that she has prepared:


"We'll restrict the 'broken bands of color' to be congruent rectangles, congruent right triangles, and congruent trapezoids. You can make copies of the given polygons and you can construct your work however you want, within those parameters. You also need to fill out this table":

	Number	Height	Base 1	Base 2	Area	Total Area
Triangles				n/a		
Trapezoids						
Rectangles				n/a		

As the students work, Ms. Richardson circulates the room, providing support as needed. If necessary, she reminds the students to use the "rigid polygon" option to make copies of each polygon, and that they can reflect and rotate them. She also suggests to some students that they might find it easier to complete the task if they use pencil and paper.

She walks over to Kevin and Maria and sees that they have not begun. Ms. Richardson points to the polygons in the file. "If you used the original dimensions for the triangles,

rectangles, and trapezoids, how could you fill the triangle?" Maria makes a few copies of the trapezoid and moves them around to fit in the triangle.
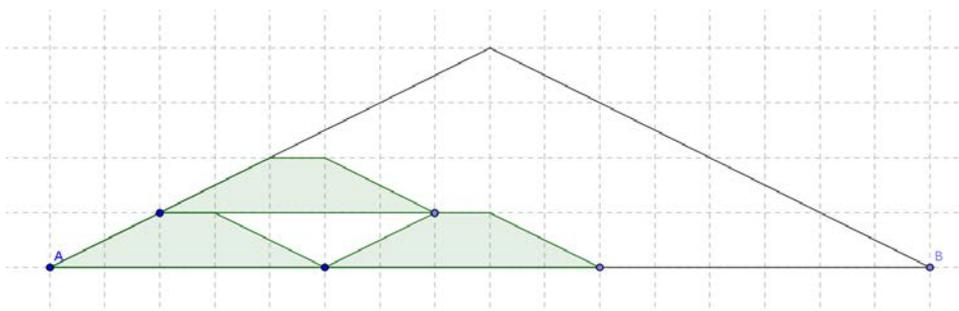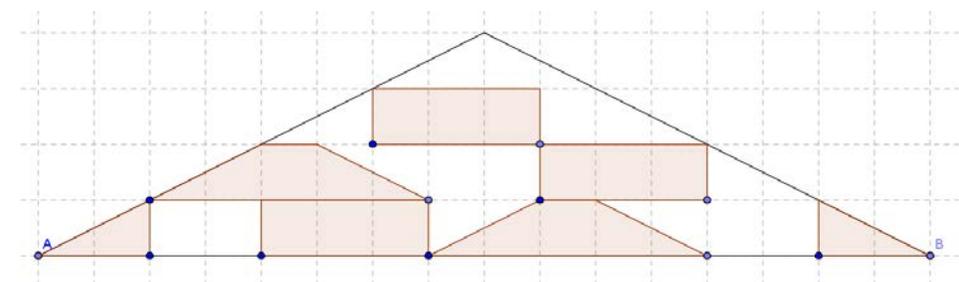

Kevin copies the triangle and moves them around to fit, rotating a few.

They continue to copy, rotate, and reflect the polygons. Ms. Richardson leaves them to complete the task.

Nathan and Zach are using triangles, rectangles, and trapezoids, but they are having difficulty identifying appropriate dimensions for the polygons. They are using a guess and check method.


Ms. Richardson asks them to pause for a moment. "How did you think about decomposing the pentagon in the opening?"

Nathan responds, "The way I thought to decompose a pentagon is to draw segments from one vertex to an opposite one. But that doesn't work for a triangle."

"That's right," Ms. Richardson says. "What happened when we decomposed a pentagon into triangles, rectangles, and trapezoids? What was your approach to those problems?"

Zach considers the question. "I thought about drawing perpendicular bisectors to see what happens. Since the pentagon is regular, using perpendicular bisectors keeps some of the symmetry." As he is speaking, Ms. Richardson pulls up the pentagons they had decomposed. Zach points to one. "Like this one. Drawing a segment across the top creates a triangle, and the bottom of the triangle and a perpendicular bisector creates two trapezoids."

Nathan continues the train of thought. "We can do something similar with this triangle to create a triangle and two trapezoids." He constructs two line segments to demonstrate his point. "The trapezoids are congruent."
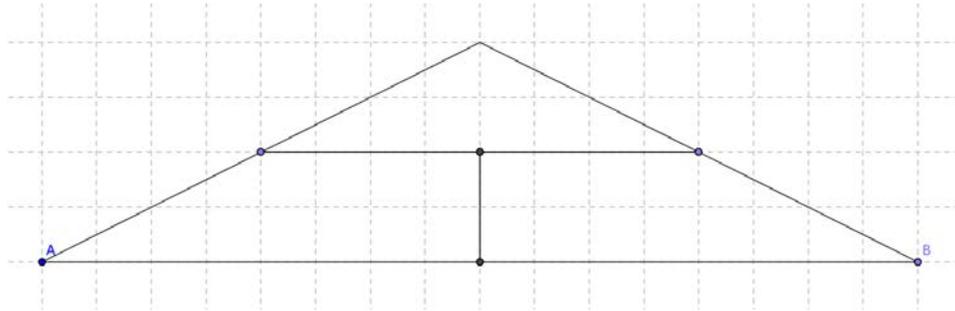
Zach also starts drawing line segments. "This looks kind of boring. If you add segments here and here, the trapezoids stay congruent and we can also add some rectangles…" Ms. Richardson walks over to another group as they continue talking.

Ms. Richardson sees that David and Emily have completed the triangle using pencil and paper. They are filling out the table but are having some difficulty.

David looks at the table, "I'm just not sure if we calculated the areas correctly. There are a lot of polygons to add up—how can we be sure it's right?" Ms. Richardson turns to Emily and asks, "How could you check your answer?"

She ponders the question, looking at their diagram. Ms. Richardson waits a moment before asking, "How did you find the area of the pentagon at the beginning of the lesson?"

"I divided it into a triangle and a trapezoid," she responds. "That made it easy, because I know how to calculate the area of a triangle and a trapezoid." She thinks for a little bit longer and then adds, "Actually, it's easy to find the area of the large triangle here—I just said I know how to calculate the area. We could add up all the areas of the polygons we divided the triangle into and then compare that to the area of the triangle. They should be equal." David begins to add the areas together while Emily works on calculating the area of the large triangle.

When students have completed decomposing the triangle into smaller polygons, she has the class come back together. Students display their work and do a gallery walk.

Closure

Ms. Richardson concludes the lesson by leading a class discussion. They note that each combination of polygons adds up to the same area (32 square units).

As an exit ticket, she asks students to calculate the area of an irregular polygon by decomposing it into triangles and other polygons.

Reflection

After class, Ms. Richardson reviews her plans for the rest of the unit. Most students have become proficient at decomposing polygons and making connections with area. She plans to introduce coordinate geometry next, and she will continue to have students use GeoGebra. While listening to the students' discussions, she noted the students who are still having difficulty with geometry. She will reconfigure the groups for the rest of the unit to optimize peer learning opportunities.