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# Lesson in Action

## Thinking Aloud: Fractions Assessment

AT A GLANCE

Launch

- Explain the purpose of the lesson, which is to review the fractions quiz using the thinking aloud strategy.
- Review thinking aloud by using a quiz problem that most students answered correctly.

Learning Task

- Direct students to work in pairs on a quiz problem that most students did not answer correctly.
- Remind students that there are technology tools available for their use.
- Circulate around the room while pairs work, listening to conversations and providing support as needed.
- Work individually with students who need additional support.
- When the students are finished, bring the class back together and have a few students demonstrate their solution methods.

Closure

- Discuss the key mathematics concepts from today's lesson.
- Have students complete an exit ticket.

## Context

Mrs. Jackson's Grade 6 class has been working on a fractions, rates, and ratios unit. Yesterday she gave the class a quiz to assess her students' understanding. After reviewing the results, she wants more detail about her students' thinking in order to effectively differentiate instruction. She decides to employ the thinking aloud strategy while reviewing the quiz problems.

## Common Core State Standards

- CCSS Math 6.RP.3 (http://www.corestandards.org/Math/Content/6/RP/A/3) Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- CCSS Math 6.NS.1 (http://www.corestandards.org/Math/Content/6/NS/A/1) Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*
- CCSS Math MP 3 (http://www.corestandards.org/Math/Practice/MP3) Construct viable arguments and critique the reasoning of others.

## Lesson Objective

Students will employ the thinking aloud strategy to reveal their understanding of ratios, rates, and the division of fractions.

## Technology

- Interactive whiteboard to communicate visually with the class
- Online version of the fractions quiz, projected onto the whiteboard to use as a reference
- Number line bars to provide a visualization option (http://nlvm.usu.edu/en/nav/frames_asid_180_g_2_t_1.html)
- Fraction bars to provide a visualization option (http://nlvm.usu.edu/en/nav/frames_asid_265_g_1_t_1.html?open=activities&from=topic_t_1.html)

# Assessment

- Observe and document students' use of the thinking aloud strategy
- Assess student understanding during the closing discussion
- Review exit tickets
- Grade 6: Focus on Fractions Quiz (http://nutmegeducation.com/registration/puww/)

# Mrs. Jackson's Class in Action

## Launch

Mrs. Jackson begins by passing back the quizzes, which she marked with comments instead of a percentage grade. "Today, we're going to review yesterday's quiz results," she says. "Almost everyone had difficulty with question 2, so that's the one we'll focus on for most of the lesson. You're going to work with your partner to solve the problem, using your thinking aloud skills."

She puts question 6—which almost everyone answered correctly—up on the interactive whiteboard:

Tori can buy five melons for $9.00. Which represents a unit rate for this situation?

a. Half a melon for $ 0.90
b. 1 melon for $1.80
c. 2 melons for $3.60
d. 10 melons for $18.00

"Before we get into our pairs to work on this problem, let's practice thinking aloud as a class with another question from the quiz," Mrs. Jackson says. "Question 6 asks what the unit rate is for Tori's situation. Can someone tell me what that means, and what information we have to solve the problem?"

Chris raises his hand. "It means you need to figure out how much one melon costs." He goes up to the whiteboard and writes:

$$5\ melons = 9\ dollars$$

"That is what we know."

"Great!" Mrs. Jackson says. "Now, who can help me think aloud about how to start solving the problem?" Jason, who rarely volunteers, raises his hand. She has Jason come up to the board.

He begins with the statement Chris wrote. "I need to figure out how many dollars one melon costs, and I know that nine dollars gets five melons. I can write this as a ratio:

$$\frac{5\ melons}{9\ dollars}$$

I know that if I multiply together with the same units, I can cancel them out. I'll try multiplying by one melon, since the answer I need has to do with one melon:

$$\frac{1\ melon}{1} \times \frac{5\ melons}{9\ dollars}$$

Jason looks at his expression a little quizzically. “That’s not right. I need to have melons in the numerator and denominator so I can cancel them out.” Several students raise their hands, but Mrs. Jackson encourages Jason to continue reasoning out loud until he realizes that the ratio can be inverted.

$$\frac{1\ melon}{1} \times \frac{9\ dollars}{5\ melons}$$

Elena had also volunteered to think aloud, so Mrs. Jackson has her finish the rest of the problem. “The melon units cancel each other out,” she says as she cancels out the units and finishes solving the problem:

$$\frac{1\ \cancel{melon}}{1} \times \frac{9\ dollars}{5\ \cancel{melons}} = \frac{9}{5} dollars$$

$$\frac{9}{5} dollars = 1\frac{4}{5} dollars = \$1.80$$

“So, this means that one melon costs $1.80,” Elena concludes.

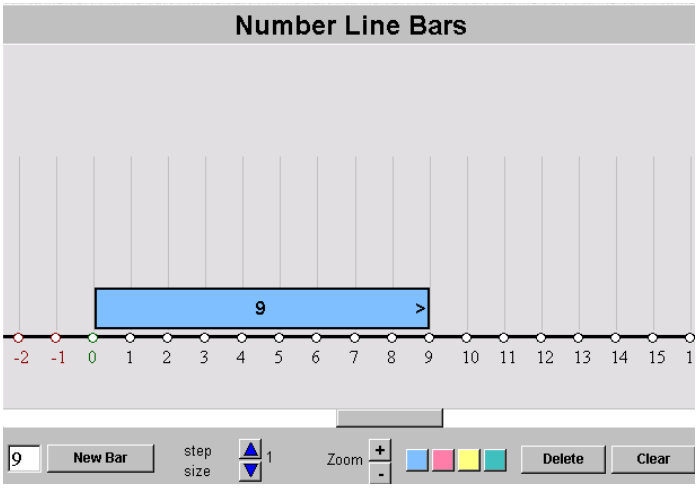
## Learning Task

As instructed, the students now move to turn to their partners and begin working on question 2 from the quiz (i.e., find the quotient; 2/3 ÷ 1/2). Mrs. Jackson reminds them, “If you find it useful, you can draw a model to help explain the problem. You can also use one of the technology tools that we’ve been using in this unit.”

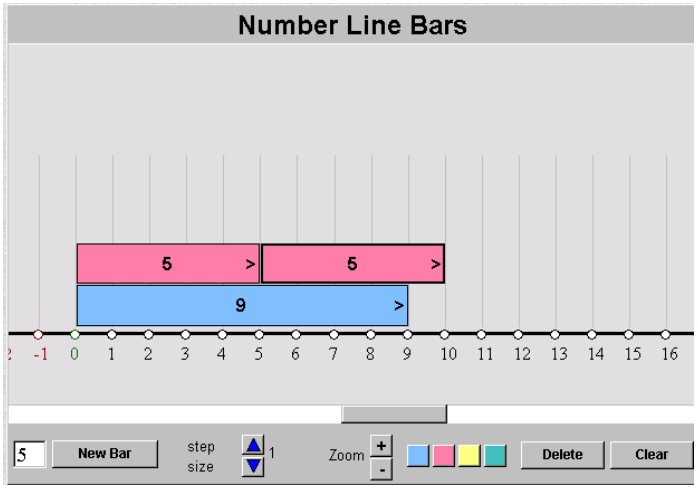
As the students begin working, Mrs. Jackson walks around and observes student discussion, offering support as needed.

Nicole is thinking aloud while drawing a model, but Jason is clearly unable to follow her reasoning. Mrs. Jackson knows that both these students are comfortable with many of the technology tools they have been using this year, and that they often benefit from visual representations. She suggests, “Why don’t you start with a simpler situation, like dividing whole numbers?” She has them go to a computer and pull up the number line bars interactive. “Jason, can you think aloud as you show how to divide nine by five?”

He begins by drawing a nine-unit bar:

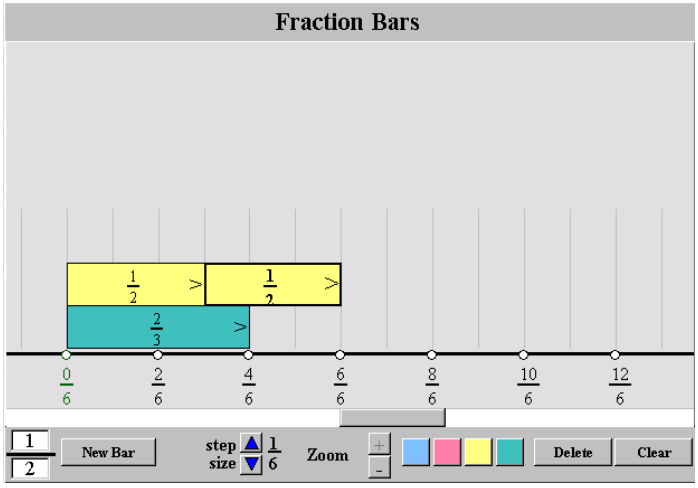


"To show nine divided by five, first draw the dividend," he begins. "Then, draw enough bars that are the length of the divisor so that it is equal to the dividend. We can't do that here," he observes, "so we'll draw just enough to be closest to nine but still greater than it."




He continues to think aloud and work with the interactive number line bars, concluding that 9 ÷ 5 = 1 and 4/5 because the 9 is "covered" by one whole 5 and 4/5 of a five, so there are 1 and 4/5 fives in one 9.

"Excellent!" says Mrs. Jackson. "Now I want you to go back to question 2, using the fraction bars interactive. This time it is Nicole's turn to think aloud." To remind her students of question 2, Mrs. Jackson writes 2/3 ÷ 1/2 on the board. As Nicole begins, Mrs. Jackson guides her to parallel Jason's thinking aloud.




Once they have set up the division problem on the interactive fraction bars, she moves on to another pair.

Chris is saying to Elena, "You get the answer by flipping the 1/2 and multiplying." Mrs. Jackson stops him, "Chris, you changed the 1/2 to two. I'm wondering—why did you choose to do that?" Chris looks at her and shrugs, "because I know it works." Looking puzzled, he seems unable to think of a reason.

Chris and Elena are fluent in both fraction multiplication and creating equivalent fractions, so Mrs. Jackson turns to Elena and asks her to write the problem in fraction form, without the division sign. Elena thinks for a moment and then writes:

$$\frac{2/3}{1/2}$$

"Ok. Now, how can you eliminate the denominator?" Mrs. Jackson asks as she points at 1/2.

Chris responds, "You turn it into one. We learned that earlier this year…." He trails off and considers his own statement. "You turn 1/2 into one by multiplying it by 2/1… and to create an equivalent fraction you multiply by another fraction that has the same numerator and denominator…" As he considers how to create an equivalent fraction, he writes:

$$\frac{2/3}{1/2} \times \frac{2/1}{2/1}$$

Elena chimes in, "Oh, I see! Now the denominator is equal to one; if we just erase it now we've created a multiplication problem."

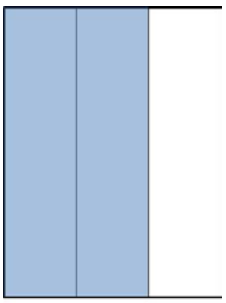
$$\frac{2}{3} \times \frac{2}{1} = \frac{4}{3}$$

Elena still looks a little puzzled. "It works for this problem, so it should work for all division problems, right? Even for a hard fraction, like 4/7?" Mrs. Jackson turns to Chris for a response. Chris says, "I don't know, I think we should experiment with more fractions to see if that's true."

Once she feels that all pairs are able to think aloud effectively, Mrs. Jackson meets individually with several students. These students need additional support and benefit from one-on-one attention, so she works with them to think aloud about specific problems they had trouble with on the quiz.

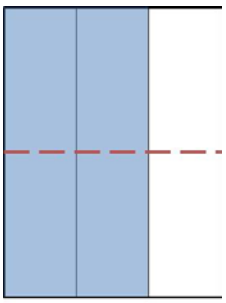
After pairs have had sufficient time to think aloud, she brings the class back together for discussion. She calls on several students to demonstrate their solution method.

Brandon demonstrates the solution that he worked on individually with Mrs. Jackson. First, he draws a model to represent 2/3:



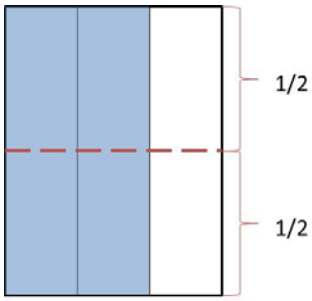
2/3

"I take a rectangle and divide it into three equal columns. The shaded part represents 2/3."



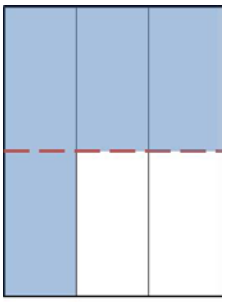
4/6

"If I draw a line across to split the rectangle into two equal parts, I've created a rectangle divided into sixths. This is still two shaded rectangles and therefore 2/3, but you can also think of it as four smaller shaded rectangles and 4/6." He points to the larger and smaller rectangles as he explains.




4/6

"The horizontal line divides the rectangle into two equal parts; each of those parts is 1/2. No matter how I move the shaded parts inside this rectangle, it will always be 4/6 of the rectangle." He moves the smaller rectangles in order to better see the number of halves.

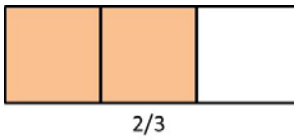


4/6

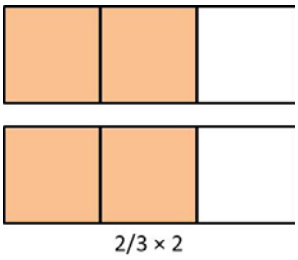
"You can see there's one 1/2 and a little more," Brandon observes. "This 'extra' rectangle is one out of three, so the number of halves is 1 and 1/3, and that's the answer."

After Brandon finishes, Mrs. Jackson comments, "Chris and Elena used equations to show that you can invert the divisor and multiply. Do you think that will work if we try that with Brandon's method?"

With prompting from the students, Mrs. Jackson draws a model similar to the one Brandon used. First she draws 2/3:




She then multiplies by the inverse of the divisor—1/2:




After some discussion, the class agrees that this gives the same answer.

## Closure

Mrs. Jackson concludes the lesson by leading a class discussion, asking students to describe the key mathematics concepts that were discussed today. She asks students to identify any misconceptions they had, and to explain how they were able to correct their thinking.

As a closing task, she has each student complete an exit ticket.

## Reflection

Later that day, Mrs. Jackson sits down and reviews the exit tickets. Almost no students were able to divide a fraction by a fraction when they took yesterday's quiz. She is pleased to see that more than half of the students succeeded on the exit ticket. During the closing discussion, several students were able to clearly explain their method. She thinks that more peer interaction would benefit many of the students who are still unsure about this concept, and about a few other unit concepts. She plans the lessons for the rest of the unit, thinking about how to differentiate instruction so that all students meet the learning goals.