



POWERUP
WHAT WORKS

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Lesson in Action

AT A GLANCE

Launch

- Introduce the lesson by reviewing terms.
- Have students do a simple practice problem, writing answers on individual whiteboards.
- Work on another practice problem as a class.
- Explain to students the importance of organizing their work.

Learning Task

- Demonstrate one method for organizing factors (factor rainbows).
- Walk students through the process of completing a factor rainbow.
- Once the factor rainbow is complete, discuss with the class how to know if it is complete.
- Demonstrate a second method for organizing factors, using an online tool.
- Have students choose a strategy and practice finding all the factors of a couple of numbers.
- Review how to find common factors, including the greatest common factor.

Closure

- Have students write a summary about what they learned in the lesson.
- Have students copy the summaries into their notebooks.
- Begin work on homework problems.
- Have students who finish early explore another virtual manipulative as an extension.

Organizing: Pairs of Factors

Context

Mr. Rutter's Grade 6 class is working on a multiplication unit that focuses on multiples and factors. In this lesson, he will focus on finding greatest common factors to prepare them for the upcoming fractions unit. Understanding these concepts is important for understanding fractions.

Common Core State Standards

- ▶ [CCSS.Math.6.NS.4](http://www.corestandards.org/Math/Content/6/NS/B/4) (<http://www.corestandards.org/Math/Content/6/NS/B/4>) Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express $36 + 8$ as $4(9 + 2)$.
- ▶ [CCSS.Math.MP7](http://www.corestandards.org/Math/Practice/MP7) (<http://www.corestandards.org/Math/Practice/MP7>) Look for and make use of structure.

Lesson Objective

Students will learn organizing tools for working with factors and multiples. The strategy of organizing allows students to work with information and numbers in many different ways.

Technology

- ▶ Interactive whiteboard to communicate visually with the class
- ▶ Calculator
- ▶ [Factorize applet](http://www.shodor.org/interactivate/activities/Factorize/) (<http://www.shodor.org/interactivate/activities/Factorize/>) to use as a computer-based alternative for organizing factors
- ▶ [Factor Tree virtual manipulative](http://nlvm.usu.edu/en/nav/frames_asid_202_g_2_t_1.html) (http://nlvm.usu.edu/en/nav/frames_asid_202_g_2_t_1.html) to use as an extension activity and reinforce knowledge of factors

Assessment

- ▶ Question students about the completeness of the list of factors
- ▶ Observe answers displayed on individual whiteboards
- ▶ Guide student summaries about organizing and finding common factors

Mr. Rutter's Class in Action

Launch

Mr. Rutter begins by reviewing the terms “factor” and “factor pair.” He then asks students to write the factors of 10 on their individual whiteboards. Almost all of them have all four factors, but a few forgot 1 and 10.

Next, Mr. Rutter asks for all the factors of 48. As students volunteer answers, he writes them on the whiteboard. Here is the list the class creates:

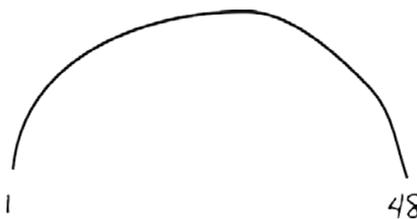
2, 24, 4, 8, 6, 1, 48, 12

“Is that all of them?” he asks. Pam suggests 3.

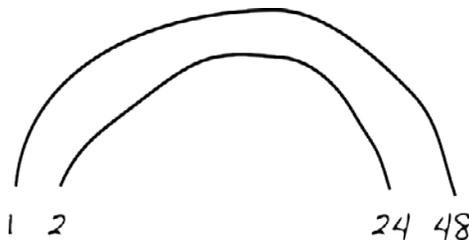
“Good,” Mr. Rutter says, “but this still isn’t all of them! We need to find a way to organize our work to be sure that we’re getting all of the factors. We can look for patterns that will help us.”

Learning Task

“I’m going to show you a couple of ways to organize factors,” Mr. Rutter continues. “First, let’s try ‘factor rainbows’ to write all the factors of 48. We’ll start with 1 times 48 is 48.” On the whiteboard, he writes 1 and 48 and then draws an arc connecting the two.



He explains, “I’m starting with 1 because it’s the smallest factor of 48, and 48 is the other half of the factor pair. I’m connecting them with an arc to show they’re part of the same factor pair.” Pointing to the space at the right of the 1, Mr. Rutter asks, “What’s the next number that’s a factor?” Several students call out “2!” and he writes 2 on the board. When he asks, “What’s the other factor in the pair?” and students respond, “24!” he writes 24 to the left of 48 and connects the two numbers with another arc.



Mr. Rutter points to the numbers on the right. “Could there be another factor between 24 and 48? Talk with a partner and come up with an explanation.”

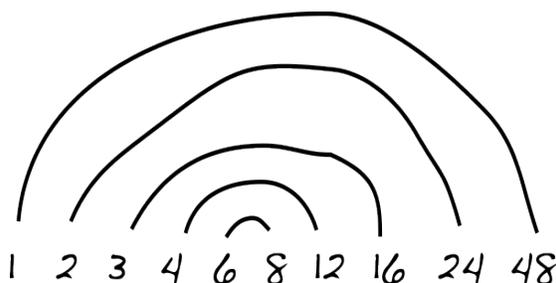
After the students talk it over for a minute, Joanna volunteers to explain. “24 is paired with 2 and 48 is paired with 1. That means that if something between 24 and 48 was a factor, it would have to be paired with a number between 1 and 2. But that’s a fraction! We can’t use fractions for factors.”

Mr. Rutter asks, “Who agrees with Joanna?” Several students disagree or are unsure, but no one has another explanation.

“Go ahead and use a calculator to figure out what factor would have to be paired with a number between 24 and 48,” Mr. Rutter says. When students finish using their calculators, they discuss the results with their partners. This time, almost everyone agrees with Joanna. Mr. Rutter notes who didn’t agree so he can check in with them later.

“Joanna is right,” he says. “Any number between 24 and 48 can’t be a factor of 48 because the other number in the pair would have to be between 1 and 2.” He makes an arc with his finger between the two arcs on the whiteboard.

Mr. Rutter continues creating the factor rainbow—skipping 5 because it’s not a factor of 48—until he ends with 6 and 8 in the middle.



“Now I’m done,” he says. “Why can I stop here?”

Mark says, “Seven isn’t a factor. Eight is the next factor, but you already have it.”

“OK,” Mr. Rutter continues, “give a thumbs up if you think I have all the factors, down if you think I don’t, and to the side if you’re not sure.” Most students have their thumbs up, but several have their thumbs to the side.

“We do have them all,” Mr. Rutter says, “but it’s OK if you weren’t sure. You can try a few of these until you get more comfortable with it. Right now, I want to show you another way to organize.”

He puts the Factorize applet up on his whiteboard and says, “I’m going to demonstrate this applet, but you can also use this method by writing down products on paper or your whiteboards.” He specifies 48 as the number to use.

“A good place to start is with the smallest factor, which is always 1. The other factor in this factor pair is 48.” He enters 1 times 48.

Get New # Use This #: 48 Show Commutative Property Do Not Show Commutative Property

Enter factorizations of 48:

Hint
That is a good factorization.

1 x 48

Mr. Rutter tells the class, “Now, we’re going to try numbers in order. We have 1, so now we’ll try 2, and 3, and so on. What’s the factor that goes with 2?” When Keisha calls out “24!” as the other factor, he enters 2 times 24.

Get New # Use This #: 48 Show Commutative Property Do Not Show Commutative Property

Enter factorizations of 48:

Hint
That is a good factorization.

2 x 24

1 x 48
2 x 24

“This applet helps by checking that the pairs are correct,” Mr. Rutter says. “If you don’t need the check, you can just write down the pairs.” He continues with the applet, until he has the following:

Get New # Use This #: 48 Show Commutative Property Do Not Show Commutative Property

Enter factorizations of 48:

Hint
That’s all the factorizations.
Now select a factorization and build it below.

1 x 48
2 x 24
3 x 16
4 x 12
6 x 8

“Now I’m done,” Mr. Rutter says. “Those are all the factorizations. I didn’t need the applet to tell me I could stop. How did I know?” Mei says, “You already have 8, just like when you stopped with the rainbow.”

Mr. Rutter agrees, “This is pretty much the same thing we did with the rainbow. Here the factors are in two columns instead of a line. In both cases, we worked up from 1, finding factors and their paired factors. We stopped when going on meant we’d be repeating numbers.”

Mr. Rutter asks the students to find all the factors of a couple of numbers, using either strategy. It doesn’t surprise him that most students prefer the factor rainbow, since most of them like more visual displays.

Next, he shows them how to use a pair of lists to find common factors for two numbers—with a special focus on the greatest common factor.

Closure

Before moving on to the homework problems, Mr. Rutter has the class summarize what they know about organizing strategies and finding common factors. He prompts them to copy the summaries into their notebooks.

He has them work individually on their homework, finding greatest common factors for several pairs of numbers.

When several students finish early, he shows them the Factor Tree virtual manipulative as an extension activity. “You’ll see the factor trees again later when we talk more about prime factors.”

Reflection

All of Mr. Rutter’s students were able to use at least one of the organizing structures. Most were then able to use those lists to find the greatest common factor and understand the idea behind the organizing structures. He wants to continue to foster that understanding as he introduces more organization structures for future topics. He notes the students who did not seem to understand how the organization worked, as well as those who struggled with using their lists to find the greatest common factor. He makes a note to check back in with each of these students to provide additional support and modeling. Mr. Rutter also records the ideas that students provided in the final summary, both for finding common factors and for organizing. He plans to he continue monitoring student progress.